Introduction to Cellular Automata (CA)

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CA origins: 1940s - 1950s

- Los Alamos Lab, Manhattan Project
- Self-replicating systems, theory of computation, early robotics
- "Robot building another robot"
- Liquid motion - group of discrete cells

John von Neumann

Stanislaw Ulam
CA basics

- discrete space, discrete time, discrete states - fully digital framework
- finite or infinite space
- finite space: fixed or self-similar boundaries
- cell neighbourhood, e.g. von Neumann’s:
- or Moore’s:
- iterative process
- state of a cells at next iteration is a function of the state of this cells and of the states of its neighbourhood cells at the current iteration
CA example: game of life

- 1970s, John Conway:

- 2 cell states: alive or dead (1 or 0), Moore’s neighbourhood

- Rules
  1. Any live cell with fewer than two live neighbours dies, as if caused by under-population
  2. Any live cell with two or three live neighbours lives on to the next generation
  3. Any live cell with more than three live neighbours dies, as if by over-population
  4. Any dead cell with exactly three live neighbours becomes a live cell, as if by reproduction

- Simple rules - very complex emergent behaviour

- about the game
CA example: game of life: "glider"

- empty cells - dead, circles - alive
- a moving pattern
  - glider gif animation
  - animations
- Gosper glider gun - glider manufacturer
- Purpose? Pretty abstract...
A quasi-CA example: 1D traffic modelling

Purpose? e.g. traffic jam or speed limit analysis

- 1D array of integers
- State, $S$, is speed: $S = 0$ - empty (no car), $S > 0$ - car with speed $S$. $S_{\text{max}}$ - max. speed. Direction is postulated.
- Neighbourhood is **not well defined**. $G$ is gap between one car and the next (cell with $S > 0$)
- Rules ( $i$ - current iteration, $i+1$ - next iteration )
  1. Acceleration: if $S(i) < S_{\text{max}}$ and $S(i) < G(i)$, then $S(i + 1) = S(i) + 1$.
  2. Slowing down: if $S(i) > G(i)$, then $S(i + 1) = S(i) - 1$
  3. Car is moving $S(i)$ cells

Example: $S_{\text{max}} = 5$, *self-similar* boundary conditions

<table>
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<tr>
<th>i</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
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<tr>
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<td>2</td>
<td>2</td>
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<tr>
<td>i+2</td>
<td>2</td>
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</tbody>
</table>

Adapted from: R. Cochinos, 1D traffic simulation
Primitive 3D solidification - probabilistic CA

- States: liquid = 0, crystals > 0.
- Cell state uniquely encodes crystal orientation tensor, i.e. a look-up table.
- Each iteration a liquid cell acquires a state of a randomly chosen neighbour (3D Moore’s neighbourhood - 26 cells).
Primitive probabilistic 3D solidification - results

For more results ▶ CGPACK
Dendritic grain growth - 2D or 3D probabilistic CA

- Growth probability:
  \[ p(\phi) = 1 - \exp\left[-\left(\frac{\phi}{\kappa}\right)^\eta\right]. \]

- Each liquid cell generates a random number \( r \in [0 \ldots 1] \).

- If \( r < p \) - cell becomes solid.

- \( \phi \) is a growth potential - depends on temperature, composition and morphology.

- Temperature field is **input** to CA! Can be directional, uniform, random or a result from another calculation, e.g. solution to a heat conduction PDE (R. E. Neapolitano, T. H. Sandlers, Jr (1998) Mat. Res. Soc. Symp. Proc. 529:101-106)

- H. K. D. H. Bhadeshia’s pages
- M. Rappaz, Ch. A. Gandin work
- M. J. M. Krane page
Recrystallisation

- The grain boundary velocity $\dot{x} = nm$, $n$ - the normal to the grain boundary segment, $m$ - mobility, $p$ - the driving force.

- If $\dot{x}\Delta t \geq c$, where $\Delta t$ - time increment, $c$ - cell size, then a cell joins the growing grain.

- Mobility strongly depends on temperature: $m \approx \alpha \exp(-\beta/T)$, $\alpha, \beta$ - some parameters, $T$ - temperature.
Lattice gas automata

- 1973, 1976 - Hardy, Pomeau and de Pazzis ("HPP rule")
- Conservation of momentum and particle number
- 2D
- Min. 4 bits per cell - constant speed.
- von Neumann’s neighbourhood
- Collision, exclusion and motion rules
- Discrete MD - much simpler micro behaviour - similar macro behaviour
- (-) Anisotropy (mesh dependence)!
Lattice gas automata examples, also on hex grid (FHP)

1986: Frisch, Hasslacher, Pomeau (FHP) - hexagonal grid - isotropic!
Lattice Boltzmann methods

- A discretised Navier-Stokes equation.
- Distribution function $f_i$ instead of discrete states, $i \in [1..6]$ - direction from one cell to another.
- Lattice Boltzmann equation:

$$f_i(x + c_i \Delta t, t + \Delta t) - f_i(x, t) = -\alpha(f_i - f_i^M)$$

- the change in the distribution function is proportional to the collision term, which is subject to a Maxwell distribution, $f_i^M$.


Wikipedia

LBM animations
Other CA examples

- Sand pile formation
- Ising magnetisation
- Land use
- Diffusion
- Fire

Fracture: \( CA + FE = CAFE \) multi-scale model

- CA (microstructure) + FE (continuum mechanics) = CAFE.
- Fracture criterion, e.g. transgranular cleavage - fracture stress or fracture strain.
- FE \( \rightarrow \) CA (localisation) - stress, strain fields
- CA \( \rightarrow \) FE (homogenisation) - damage variables
Wolfram - CA for everything

Stephen Wolfram  ▶️ stephenwolfram.com

► A genius - 1st paper at 15, PhD at 20, ”New kind of science”, Mathematica, Wolfram alpha, etc.
► CA underpins nature: physics, biology, medicine, chemistry, society, human condition, future of history...
► Space, time, universe are governed by CA rules
► Simple rules lead to seemingly unbounded complexity - computational universe
► mad?
CA summary

- Deterministic or probabilistic
- 1D, 2D, 3D or nD
- Fixed, self-similar (periodic), adiabatic, etc. boundaries
- Integer or real states
- Mostly (always?) local influence
- Time and length scale independent
- Problem agnostic (physics, society, biology, etc.)
- Fixed regular Eulerian mesh (grid)
- Similar to Monte-Carlo and phase field models
- Simple rules but complex emergent behaviour
- Simple rules = fast computation, no equilibrium or convergence checks
Books